

Keeping the Devil in the Details: A Feasible Approach to Aggregating Trade Distortions

by

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Abstract

We analyze the properties of tariff revenue and expenditure aggregators - tariff aggregators that keep expenditure, respectively tariff revenue constant. We derive key theoretical properties of these tariff aggregators, and under certain assumptions, develop closed-form solutions, promoting the use of these tariff aggregators in empirical research.

Keywords: tariff aggregators, expenditure aggregator, tariff revenue aggregator

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1. Introduction

An important challenge in applied international economics is selecting the appropriate method of aggregation of thousands of tariff lines from a country's tariff schedule to match the higher-level aggregation data available for production/consumption. For example, an ad-hoc method of tariff aggregation, like trade-weighted average, may severely underestimate the global benefits of EU agricultural trade reform - Martin et al. (2003) show that these gains are 150 percent higher when appropriate tariff aggregates¹ are used. New approaches, with rigorous theoretical foundations for the aggregation problem have emerged. Anderson and Neary (1993) proposed a uniform tariff that yields the same welfare as the original differentiated tariff structure. Similarly, Bach and Martin (2001) proposed two new tariff aggregators that keep expenditure, respectively tariff revenue constant, as the center of a compensating variation approach to consistent aggregation. However, their approach didn't provide a closed-form solution for the tariff revenue aggregator, rendering the proposed aggregator cumbersome to use in applied work. Further, they didn't analyze the theoretical properties of these aggregators to provide insights into their potential behavior. This paper improves upon Bach and Martin (2001) by deriving the properties of the proposed aggregators, which significantly increases the applicability of these aggregators in both empirical economics and policy simulation models. In particular, we show that in the case of their tariff revenue aggregator there may be *multiple* uniform tariffs that yield the same tariff revenue. To deal with this significant challenge we properly define the tariff revenue aggregator so that it leads to a *unique* solution. Furthermore, we derive key theoretical properties of these tariff

¹ They use in an empirical application the expenditure and tariff revenues aggregators whose theoretical properties are derived in this paper.

aggregators, and under common assumptions in applied trade, we develop *closed-form* solutions for the expenditure and tariff revenue aggregators, which allows for practical applications of these aggregators in empirical trade. Finally, we explore the relations between the trade-weighted average tariff, the expenditure aggregator and the revenue aggregator.

2. Model

Bach and Martin (2001) assume that the structure of a competitive, small open economy can be captured by an income-expenditure condition,

$$(1) \quad e(\mathbf{p}, u) - r(\mathbf{p}, \mathbf{v}) - (\mathbf{e}_p - \mathbf{r}_p)(\mathbf{p} - \mathbf{p}^w) - f = 0$$

and the vector of behavioral equations²,

$$(2) \quad e_p(\mathbf{p}, u) - r_p(\mathbf{p}, \mathbf{v}) = \mathbf{m}$$

where $e(\mathbf{p}, u)$ is the expenditure function of the representative household, \mathbf{p} is a given vector of domestic sectoral price aggregates, u is domestic utility, $r(\mathbf{p}, \mathbf{v})$ is domestic revenue from production, and \mathbf{v} is a vector of productive resources; \mathbf{m} is the vector of imports, and f is the exogenously-determined net financial inflow from abroad.

Based on equation (1) and considering prices \mathbf{p} and the level of utility u^0 as exogenous, the balance-of-trade function \mathbf{B} can be written as:

$$(3) \quad \mathbf{B}(\mathbf{p}, u^0) = e(\mathbf{p}, u^0) - r(\mathbf{p}, \mathbf{v}) - (\mathbf{e}_p - \mathbf{r}_p)(\mathbf{p} - \mathbf{p}^w) - f$$

In the rest of the paper we base our analysis on the assumption that the conditions proposed by Bach and Martin (2001) hold. In this framework, let e be the expenditure function, $e(\mathbf{p}_j, u_j^0)$, where \mathbf{p} is the vector of domestic price for goods and u^0 is the utility level associated with consumption of goods. The goods can be divided into domestically produced goods - with price \mathbf{p}^d , and imported goods - with domestic price \mathbf{p}^t , so the complete domestic vector price may

² We use bold letters for vectors.

be written as $\mathbf{p} = (\mathbf{p}^d, \mathbf{p}^t)$, where the disaggregated tariffs enter the definition via domestic prices of imported goods - \mathbf{p}^t .

Bach and Martin (2001) define the tariff aggregator for expenditure as the uniform tariff, τ^e , which requires the same level of expenditure on imported commodities as the observed vector of tariffs to maintain utility level u^0 :

$$(4) \quad \tau^e = [\tau^e \mid e(\mathbf{p}^d, \mathbf{p}^w(1+\tau^e), u^0) = e(\mathbf{p}^d, \mathbf{p}^t, u^0)]$$

A tariff revenue aggregator may be defined as the uniform tariff that yields the same tariff revenue as the observed vector of disaggregated tariffs, conditional on the utility level underlying the expenditure function and the resource endowments underlying the domestic revenue function (Bach and Martin, 2001):

$$(5) \quad \tau^R = [\tau^R \mid tr(\mathbf{p}^w(1+\tau^R), \mathbf{p}^w, u^0, v^0) = tr(\mathbf{p}^t, \mathbf{p}^w, u^0, v^0)]$$

3. Properties of the aggregators

The definitions given in equations (4) and (5) do not guarantee existence, uniqueness or economic meaning for the proposed tariff aggregators. In the rest of this paper we consider a Constant-Elasticity of Substitution (CES) functional form for the expenditure function and for the import demand functions. With this functional form, the expenditure function and the tariff revenue function are:

$$e = \left(\beta^d p^d + \sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} u^0$$

$$tr = \sum_i \beta_i \left(\frac{p}{p_i} \right)^\sigma (p_i - p_i^w) u^0$$

where $p = \left(\beta^d p^d + \sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is a price index, p^d is the price of the domestic good

and p_i and p_i^w are the domestic and world prices of the import good i , the parameters

$\beta^d, \beta_1, \beta_2, \dots$ are the expenditure shares (domestic and import). By appropriate selection of the units of measurement, all domestic prices may be set equal to 1 in the base equilibrium and, in consequence, $p_i^w = 1/(1 + \tau_i)$, where τ_i is the *ad-valorem* tariff for good i . In this context, Bach and Martin (2001) showed that the expenditure tariff aggregator has the following closed-form form:

$$(6) \quad \tau^e = \left(\frac{1 - \beta^d}{\sum_{i=1}^n \beta_i (1 + \tau_i)^{-(1-\sigma)}} \right)^{\frac{1}{1-\sigma}} - 1$$

This closed-form solution guarantees the existence and uniqueness of the expenditure tariff aggregator. We prove that the expenditure aggregator is always positive when $\sigma > 1$, meeting this basic criterion for economic relevance.

Proposition 1. For $\sigma > 1$, τ^e is positive.

Proof: From (6) and with the domestic prices set to 1 in the base equilibrium, world prices are

$$p_i^w = 1/(1 + \tau_i) \text{ and we see that } \sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma} = \sum_{i=1}^n \beta_i (1 + \tau_i)^{\sigma-1} > \sum_{i=1}^n \beta_i \text{ as long as there is}$$

at least one positive tariff. As the denominator is $1 - \beta^d = \sum_{i=1}^n \beta_i$, the ratio in parentheses in (6)

is less than 1. The ratio to a negative power is greater than one, so τ^e is positive.

The tariff revenue index, τ^R can be obtained by setting the tariff revenue function (5) equal to the corresponding expression with a uniform tariff, and solving for τ^R . This is similar to solving:

$$(7) \quad c = h(\tau^R)$$

where $h(\cdot; \mathbf{p}_j^w, u_j^0, v_j^0)$ is a real function of τ^R and c depends on disaggregated tariffs.

Proposition 2. For equation (7), for certain values of the parameter c , there are at least two solutions.

Proof: We sketch the ideas first, and follow with a more detailed exposition.

The function $h(\cdot)$ is a continuous and positive function for positive tariffs, with $h(0)=0$, $\lim_{t \rightarrow \infty} h(t) = 0$ and a maximum at M . For any value $c < M$ we may apply the intermediate value property and find at least two solutions.

Figure 1. Uniform tariff solutions for constant tariff revenue

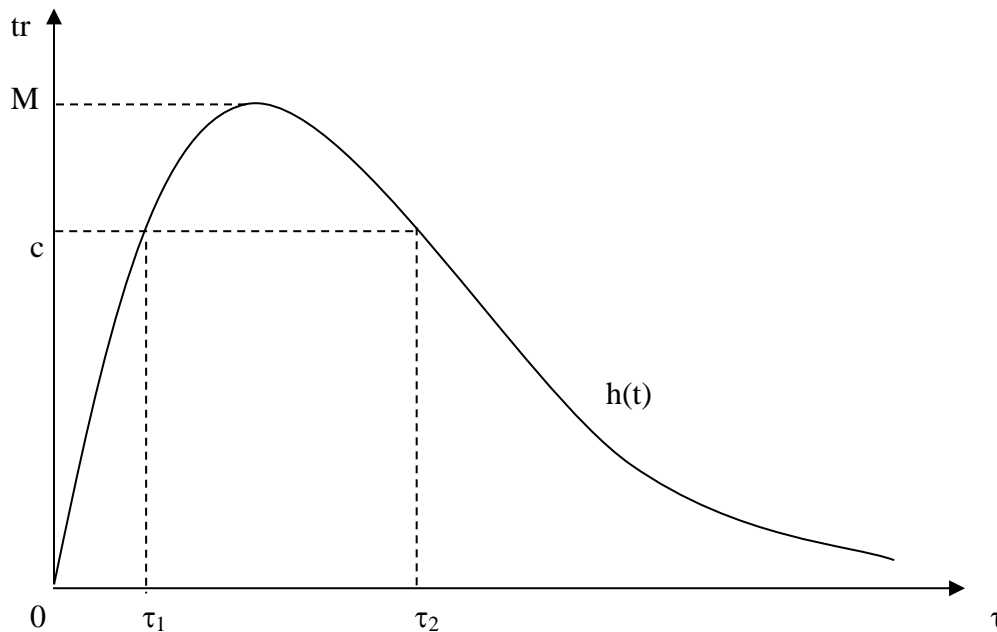


Figure 1 is the (in)famous Laffer Curve for tariff revenues. Tariff revenues increase with the increase of tariffs, up to some point M . Higher tariffs would decrease revenues, because the reductions in import volumes associated with increased tariffs outweigh the revenue gains. If $c < M$, there are two values τ_1 and τ_2 such that $c = h(\tau_1) = h(\tau_2)$. However, only one of these tariff rates is in the economically relevant range. No well-informed government would set tariff rates beyond the revenue-maximizing level. Assuming that the objective is to keep tariff revenue constant and assuming economic rationality, we use the lowest uniform tariff that keeps tariff

revenue constant. This aggregator has the property that tariff revenues are increasing in this range.

For a detailed proof, we write an explicit form for equation (7):

$$(8) \quad \frac{\sum_{i=1}^n \beta_i p_i^W \tau_i}{\sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma}} = \tau^R (1 + \tau^R)^{-\sigma} \left[\beta^d + (1 + \tau^R)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}$$

We define the function $h: R^+ \rightarrow R^+$, $h(t) = t(1+t)^{-\sigma} \left[\beta^d + (1+t)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}$. The

function h can be written as $h(t) = k(t) * m(t)$, with $k(t) = t(1+t)^{-\sigma}$ and

$m(t) = \left[\beta^d + (1+t)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}$. The function $k(t)$ has the following properties:

1. $k(0) = 0$
2. $\lim_{t \rightarrow \infty} \frac{t}{(1+t)^\sigma} = \lim_{t \rightarrow \infty} \frac{1}{\sigma(1+t)^{\sigma-1}} = 0$, from l'Hôpital's rule.
3. $k'(t) = \frac{1 - (\sigma - 1)t}{(1+t)^{\sigma+1}}$, the derivative being positive for $t < \frac{1}{\sigma - 1}$, zero for $t = \frac{1}{\sigma - 1}$ and negative for $t > \frac{1}{\sigma - 1}$.

The function $k(t)$ starts from zero, increases until it reaches the maximum in $t = 1/(\sigma - 1)$ after which it decreases, converging asymptotically to zero.

The function $m(t)$ has the following properties:

1. $m(0) = \left[\beta^d + \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}$ with $m(0) > 0$. Similar with the proof of Proposition 2,

$$\beta^d + \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} > 1 \text{ so } m(0) < 1.$$

$$2. \lim_{t \rightarrow \infty} m(t) = (\beta^d)^{\frac{\sigma}{1-\sigma}}. \text{ As } \beta^d < 1, \lim_{t \rightarrow \infty} m(t) > 1.$$

$$3. m'(t) = \frac{\sigma}{1-\sigma} \left[\beta^d + (1+t)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}-1} \sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma} (1-\sigma)(1+t)^{-\sigma} > 0, \text{ so}$$

$m(t)$ is an increasing function.

Note that the function $h(t)$ is a continuous and positive function with $h(0)=0$, $\lim_{t \rightarrow \infty} h(t) = 0$, has a

maximum on its domain and it reaches this maximum M . For any tariffs, imports and domestic

consumption such that $c = (\sum_{i=1}^n \beta_i p_i^w \tau_i) / (\sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma}) < M$ we may apply the intermediate

value property and find at least two solutions.

Observation. The definition of the tariff revenue aggregator may be amended as follows: In any case where there are two feasible solutions for this aggregator, the tariff revenue aggregator may be defined as **the lower valued** uniform tariff that will yield the same tariff revenue as the observed vector of disaggregated tariffs.

4. Separability between domestic and imported goods

With the additional assumption of separability between domestic and imported products – an assumption that is made routinely in CGE models – the aggregators can be applied only over foreign products. This specification requires the usual assumptions for such two-stage budgeting, such as weak separability in demand and homotheticity of the sub-utility functions at the lower level of nesting, but these assumptions are virtually universal in trade applications. Therefore, we can rewrite the expenditure function and the tariff revenue function:

$$e = \left(\sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} u^0$$

$$tr = \sum_i \beta_i \left(\frac{p}{p_i} \right)^{\sigma} (p_i - p_i^w) u^0$$

where $p = \left(\sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is a price index and the variables were described in section 3.

These assumptions (and functional forms) hold for the rest of the paper.

4.1 The Expenditure Tariff Aggregator

Based on the above assumptions (6) leads to:

$$(9) \quad \tau^e = \left(\sum_{i=1}^n \beta_i (1 + \tau_i)^{\sigma-1} \right)^{\frac{1}{\sigma-1}} - 1$$

4.2 The Tariff Revenue Aggregator

The tariff revenue aggregator, τ^R , can be obtained by setting the left-hand side in (5) equal to the corresponding expression with a uniform tariff, and solving the equation for τ^R . Therefore, a tariff aggregator ranging over the imported goods alone (with $p_i = (1 + \tau^R)/(1 + \tau_i)$ and $p_i^w = 1/(1 + \tau_i)$) can be defined and the left-hand side of the equation (5) will be:

$$(10) \quad \sum_i \beta_i \left(\frac{\left(\frac{\sum_j \beta_j}{(1 + \tau_j)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} (1 + \tau^R)}{\frac{1 + \tau^R}{1 + \tau_i}} \right)^{\sigma} \left(\frac{1 + \tau^R}{1 + \tau_i} - \frac{1}{1 + \tau_i} \right) u^0 = \tau^R \left(\sum_i \beta_i (1 + \tau_i)^{\sigma-1} \right)^{\frac{1}{1-\sigma}} u^0$$

Consequently, for the right-hand side of the equation (5) we have $p_i = 1$ and $p_i^w = 1/(1 + \tau_i)$, therefore the price index $p = 1$ and the right-hand side of the equation (5) will be

$$\sum_{i=1}^n \beta_i \tau_i / (1 + \tau_i) u^0.$$

From (10) and the above expression, we obtain a closed-form solution for τ^R :

$$(11) \quad \tau^R = \frac{\sum_{i=1}^n \beta_i \tau_i / (1 + \tau_i)}{\left(\sum_{j=1}^n \beta_j (1 + \tau_j)^{\sigma-1} \right)^{\frac{1}{1-\sigma}}}$$

where the β_j 's are value shares of imports at domestic prices ($\beta_j = M_j(1 + \tau_j) / \sum_k M_k(1 + \tau_k)$, where M_k is the value of imports of product k at border prices, and τ_k is the tariff on product k), p_j^w is the world price for product j , where ($p_j^w = 1/(1 + \tau_j)$) because the domestic price is unity, and σ is the elasticity of substitution.

Lemma. Consider x_1, x_2, \dots, x_n n positive real numbers not all equal and w_1, w_2, \dots, w_n n positive real numbers such that $\sum_{i=1}^n w_i = 1$. For r a positive real number, we define the r -weighted mean of

the x_1, x_2, \dots, x_n numbers as $A_r = \left(\sum_{i=1}^n w_i x_i^r \right)^{\frac{1}{r}}$. For the given weights w_1, w_2, \dots, w_n the r -

weighted mean of the x_1, x_2, \dots, x_n numbers has the following properties:

1. If $r > s$ then $A_r > A_s$.
2. $\lim_{r \rightarrow 0} A_r = x_1^{w_1} x_2^{w_2} \dots x_n^{w_n}$.

Proof: see Qi et al. (2000).

We use the notation t_{wa} for the weighted average tariff ($t_{wa} = \sum_{i=1}^n w_i \tau_i$), where

$$w_i = M_i / \sum_{k=1}^n M_k, \quad i = 1, \dots, n \text{ are the weights.}$$

Proposition 3. For $\sigma > 0$, $\tau^e \geq t_{wa}$.

Proof. From Lemma, we define $A_{1-\sigma} = \left(\sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ as the $1-\sigma$ weighted mean of world

prices $p_1^w, p_2^w, \dots, p_n^w$ with the weights $\beta_1, \beta_2, \dots, \beta_n$. For the same tariff for all goods, $\tau^e = t_{wa}$.

Otherwise, we use Lemma and, as $1-\sigma < 1$, then $A_{1-\sigma} < A_1$:

$$(12) \quad \left(\sum_{j=1}^n \beta_j (p_j^w)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} < \sum_{i=1}^n \beta_i p_i^w$$

We can write:

$$\sum_{i=1}^n \beta_i p_i^w = \sum_{i=1}^n \frac{M_i (1 + \tau_i)}{\sum_{k=1}^n M_k (1 + \tau_k)} \frac{1}{1 + \tau_i} = \sum_{i=1}^n \frac{M_i}{\sum_{k=1}^n M_k (1 + \tau_k)} = \frac{\sum_{i=1}^n M_i}{\sum_{k=1}^n M_k + \sum_{k=1}^n M_k \tau_k} = \frac{1}{1 + t_{wa}}$$

and (12) becomes:

$$\left(\sum_{j=1}^n \beta_j (p_j^w)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} < \frac{1}{1 + t_{wa}}$$

and further:

$$(13) \quad \tau^e = \left(\sum_{j=1}^n \beta_j (1 + \tau_j)^{\sigma-1} \right)^{\frac{1}{\sigma-1}} - 1 > t_{wa}$$

Proposition 4. For $\sigma > 0$, $\tau^e \geq \tau^R \geq t_{wa}$.

Proof. Using (9) and (11):

$$(14) \quad \tau^R = (\tau^e + 1) \sum_{i=1}^n \beta_i p_i^w \tau_i$$

Further:

$$\sum_{i=1}^n \beta_i p_i^w \tau_i = \sum_{i=1}^n \frac{M_i(1+\tau_i)}{\sum_{k=1}^n M_k(1+\tau_k)} \frac{\tau_i}{1+\tau_i} = \sum_{i=1}^n \frac{M_i \tau_i}{\sum_{k=1}^n M_k(1+\tau_k)} = \frac{\sum_{i=1}^n M_i \tau_i}{\sum_{k=1}^n M_k + \sum_{k=1}^n M_k \tau_k} = \frac{t_{wa}}{1+t_{wa}}$$

Therefore:

$$(15) \quad \tau^R = (\tau^e + 1) \frac{t_{wa}}{1+t_{wa}}$$

From Proposition 3:

$$(16) \quad \tau^e + 1 \geq 1 + t_{wa} \Rightarrow \tau^R = (\tau^e + 1) \frac{t_{wa}}{1+t_{wa}} \geq t_{wa}$$

Again Proposition 3:

$$\tau^e \geq t_{wa} \Leftrightarrow 1 - \frac{1}{1+\tau^e} \geq 1 - \frac{1}{1+t_{wa}} \Leftrightarrow \frac{\tau^e}{1+\tau^e} \geq \frac{t_{wa}}{1+t_{wa}} \Leftrightarrow \tau^e \geq (\tau^e + 1) \frac{t_{wa}}{1+t_{wa}} = \tau^R$$

From (16) and above:

$$\tau^e \geq \tau^R \geq t_{wa}$$

5. Conclusions

We analyze the tariff aggregators proposed by Bach and Martin (2001) and show that for the tariff revenue aggregator there are multiple uniform tariffs that yield the same revenue over a large set of parameters. We consequently redefine the tariff revenue aggregator so that it leads the existence and the uniqueness of the tariff aggregators. We prove that the expenditure aggregator is always positive, therefore is economically relevant. We derive the theoretical properties of the expenditure and tariff revenue aggregators, and under certain assumptions, we develop closed form solutions for both the expenditure as well as the tariff revenue aggregators, promoting the use of these tariff

aggregators in empirical research. Finally, for a positive elasticity of substitution between products, we show that the trade-weighted average tariff is lower than tariff revenue aggregator and the latter is lower than the expenditure aggregator.

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